

# Quantum-Relay-Assisted Key Distribution over High Photon Loss Channels

An-Ning Zhang,<sup>1</sup> Yu-Ao Chen,<sup>2</sup> Chao-Yang Lu,<sup>1</sup> Xiao-Qi Zhou,<sup>1</sup>

Zhi Zhao,<sup>2</sup> Qiang Zhang,<sup>1</sup> Tao Yang,<sup>1</sup> and Jian-Wei Pan<sup>1,2</sup>

*<sup>1</sup>Department of Modern Physics and Hefei National  
Laboratory for Physical Sciences at Microscale,  
University of Science and Technology of China,  
Hefei, Anhui 230026, People's Republic of China  
<sup>2</sup>Physikalisches Institut, Universität Heidelberg,  
Philosophenweg 12, 69120 Heidelberg, Germany*

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## Abstract

The maximum distance of quantum communication is limited due to the photon loss and detector noise. Exploiting entanglement swapping, quantum relay could offer ways to extend the achievable distance by increasing the signal to noise ratio. In this letter we present an experimental simulation of long distance quantum communication, in which the superiority of quantum relay is demonstrated. Assisted by quantum relay, we greatly extend the distance limit of unconditional secure quantum communication.

In recent years, most of the significant experimental advances achieved in the field of quantum communication (QC) are based on the use of photonic channels. However, serious problems occur in long distance case. Purification [1, 2] has been proposed to regain a high degree of entanglement which is normally decreases exponentially with the length of the connecting channel, owing to unavoidable decoherence in the QC channel. However, more serious problem is caused by combination of exponential losses of the photons and the dark count of the detectors which limits today's fiber-based QC systems to operation over the order of 100 kilometers [3, 4]. The losses by themselves only reduce the bit rate which is also exponential with distance. With perfect detectors the distance would not be limited. However, because of the dark counts, each time a photon is lost there is a chance that a dark count produces an error. Hence, when the probability of a dark count becomes comparable to the probability that a photon is correctly detected, the signal-to-noise ratio (SNR) tends to zero.

Quantum repeaters [5] that combines entanglement purification [1, 2], entanglement swapping [6] and quantum memory have been proposed to overcome these difficulties. Although significant experimental advances in this direction [7, 8, 9] have been achieved, a full quantum repeater has not yet been realized with current technology. Fortunately, a much simpler scheme comes as a surprise, which is now commonly called as a "quantum relay" [3, 10]. By making use of entangled photons and entanglement swapping, it works similarly as a quantum repeater, but is much closer to be implemented since it does not need the ability to store photons. Although a quantum relay does not avoid the exponential loss of signal in the communication channel, it does increase the SNR and thus can be used to extend the communication distance.

Previous work along this line includes theoretical investigations of why and how entanglement swapping helps enhance the maximum range of QC [3, 10, 11, 12], a proof-of-principle experimental demonstration of entanglement swapping [17], an experimental long-distance (2.2km, optical fiber) entanglement swapping in a quantum relay configuration [18], in which the main effort was devoted to preserve the indistinguishability of the two photons involved in the Bell state measurement (BSM), and more recently a long distance (2.2km, optical fiber) experimental entanglement swapping with a visibility high enough to violate a Bell inequality [19]. Impressive as it is, there is no experimental work to study the effect of quantum relay yet. It would be highly desirable to see in a practical way whether or not and, to

what extent, quantum relay will show its advantage on extending the available distance of QC.

In this letter, we report an experimental simulation of long distance quantum communication based on quantum relay. The interest of quantum relay is shown by performing quantum key distribution (QKD) over a high photon loss channel that a secure BB84 protocol [13] cannot be realized. To do QKD over a longer distance with the current technology, two-way classical communication could be a good choice [14, 15, 16]. The threshold of quantum bit error rate (QBER) of unconditional secure BB84 protocol extends to 20.0% [15] by two-way classical communication. In our experiment, the superiority of long-distance distribution of entangled photons assisted with entanglement swapping over the way of direct transmission is shown by extending the absolutely secure channel length to about two times.

Let us first review the salient feature of the quantum relay. Without going into complicated calculations [12], we only give the essence and main results here. As shown in Fig. 1(a), the simplest way to distribute an entangled state is to directly send a photon from a sender to a receiver, say Alice and Bob. At short distance, the visibility is high because dark counts are negligible. However, as the distance increases, more and more photons will be absorbed while the dark counts will remain constant, giving a lower SNR and thus the maximal distance for a given fidelity is limited. The implementation of a quantum relay using quantum nondemolition (QND) measurement is illustrated in Fig. 1(b). The basic idea about quantum relay is to verify at some point along the channel whether the photon is still here [10]. If the photon is still present, a classical signal indicating this fact is sent to Bob. If a photon is not detected, which means that the correct photon has been lost in its way, and then a classical signal tells Bob not to accept any output from his detectors. As a result, the limiting effect of the detector dark count can be essentially suppressed.

Using the simple linear optical device, we could conveniently implement a probabilistic QND measurement by adopting the scheme of quantum teleportation [20, 21] as shown in Fig. 1(c). Resources needed are the maximally entangled photon pairs and the polarizing beam splitter (PBS), which serves as a probabilistic Bell state analyzer [22]. It is easily seen that if any one of the two incoming photons is absent, a coincidence of the two detectors behind the PBS never happen. However, if both of them are still present, BSM will probabilistically succeed and faithfully transfer the quantum state onto the photon towards Bob. Taking into account of realistic parameters including the loss of the channel, the detector

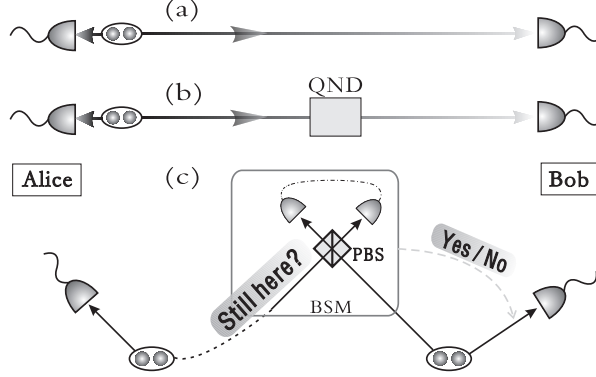


FIG. 1: Graphical show of working principle of quantum relay. (a) Entangled state distribution by directly sending a photon. (b) Quantum relay works by exploiting QND measurement to verify at some point whether the photon is still here. (c) Entanglement swapping is employed to implement a probabilistic QND measurement.

dark counts etc., numerical simulations on the performance of quantum relay have been presented in Ref. [10, 12] to show the advantage of using quantum relay.

A realistic experimental demonstration of the superiority of quantum relay will take many efforts, e.g. to distribute photons over very long distance and to implement successful BSM. Here we avoid these complexities by performing a simulative experiment. In our experiment we aim to compare the performance of QKD over high photon loss channel with and without quantum relay. We examine under the scenario that the photons are emitted from a perfect entangled photon source (PEPS), that is, exactly one photon pair per pulse. Then we equivalently consider the actual probabilistic photon source produced by parametric down-conversion (PDC) as like the photons from the PEPS have passed a certain distance in a lossy channel, during which the photons are probabilistically lost. Similarly, additional attenuation plates placed into the light path are also used to simulate the distances. Sum up the above two effects and we get the total attenuation. By using these reasonable equivalences, this experiment is greatly simplified but does not prevent us from showing its physical essence.

In the experiment, we relate the QBER to the SNR and use the QBER to characterize the performance of a quantum relay. A schematic drawing of our experimental setup is shown in Fig. 2. The required photon pairs are produced via Type-II PDC and prepared in the state  $|\Psi^-\rangle_{ab} = |H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b$  with a high SNR of 30:1 in the  $+/-$  basis, where

$|\pm\rangle = |H\rangle \pm |V\rangle$  (coefficients omitted).

In our experiment, we first test the performance of direct entangled photons distribution without assistance of quantum relay. Using the experimental setup shown in Fig. 2(a), we employ entangled photon pair 1-2 to implement quantum key distribution. We place in the light path a series of attenuation plates with a transmission rate range from  $t_a = 0.27$  to  $t_a = 6 \times 10^{-3}$  which, together with the equivalent attenuation effect from probabilistic PDC photon source  $t_s = c/(76M \times \eta^2)$ , constitute the total attenuation  $t = t_s \times t_a^2$ . Here  $c = 2.4 \times 10^4 s^{-1}$  is the two-fold coincidence rate,  $\eta = 0.15$  is the average detection efficiency.

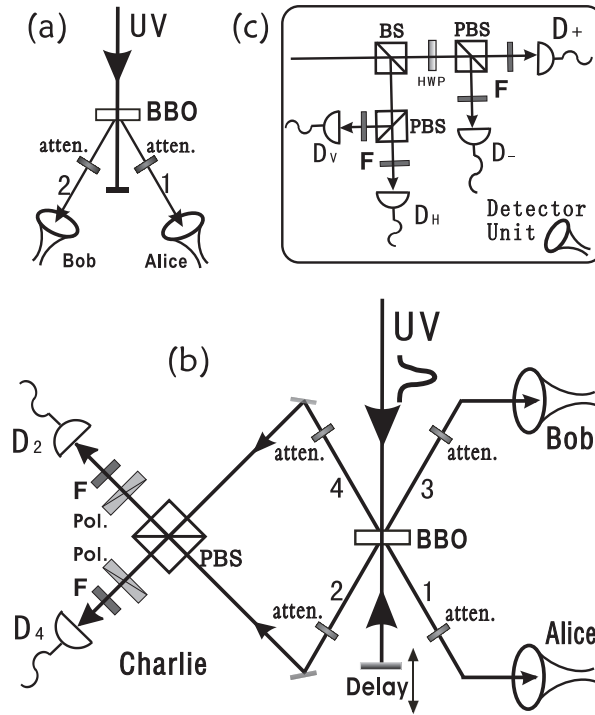


FIG. 2: Experimental setup for simulation of long distance quantum cryptography. An ultraviolet pulsed laser from a mode-locked Ti:sapphire laser (center wavelength  $394nm$ , pulse duration  $200fs$ , repetition rate  $76MHz$ ) passed through  $\beta$ -barium borate (BBO) crystal twice to generate two maximally entangled photon pairs in modes 1-2 and mode 3-4. In the experiment, we managed to obtain an average twofold coincidence rate of  $2.4 \times 10^4 s^{-1}$  with an average pump power of  $470mW$ . (a) Entangled photons are distributed by direct transmission. Attenuation plate (atten.) is inserted to simulate the distance. (b) Entangled state distribution assisted with another EPR pair (3-4) and entanglement swapping. (c) Photon detector unit for QKD.

In order to experimentally demonstrate the BB84 protocol, both Alice and Bob need to perform a polarization measurement on their own photons by randomly choosing  $H/V$  or  $+/-$  polarization basis. To achieve the random choice of the measurement basis, we let the photon pass through a 50-50 beam splitter (BS) as in Fig. 2(c). In one of the two BS outputs a polarizing beam splitter is used to perform the  $H/V$  polarization analysis. In another output of the BS, a half-wave plate (HWP) is put in front of the PBS and oriented at  $22.5^\circ$  to measure the photon along the  $+/-$  polarization basis. From the experimental results as shown in Fig. 3 (triangle dots), we can see that as the equivalent distance increases, the QBER arises dramatically. At the attenuation of about  $37.5dB$ , the QBER reaches 20.0%, a security bound of quantum key distribution [15], indicating that over attenuation condition, the unconditional security can not be guaranteed by two-way classical communication.

To accomplish the secure QKD task over the channel attenuation limit of  $37.5dB$ , we now take the advantage of quantum relay. In the experiment, another pair of EPR photons required is produced via the PDC process by the UV pulse after its reflection, whose two-fold coincidence rate is adjusted to be less than 5% different from the pair 1-2. We then superpose the photons 2 and 4 at the PBS to implement the BSM. Their path lengths are adjusted such that they arrive simultaneously. To achieve good spatial and temporal overlap, the outputs are spectrally filtered ( $\Delta\lambda = 2.8nm$ ) and monitored by fiber-coupled single-photon detectors. These processes effectively erase any possibility of distinguishing the two photons and thus lead to interference. Conditioned on the coincidence of Detector 2 and 4 behind the PBS, the entanglement of photon 2 and 4 is swapped to the photon 1 and 3 (coefficients omitted):

$$\begin{aligned}
& (|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2) \otimes (|H\rangle_3|V\rangle_4 - |V\rangle_3|H\rangle_4) \\
& \rightarrow |H\rangle_1|V\rangle_2|H\rangle_3|V\rangle_4 + |V\rangle_1|H\rangle_2|V\rangle_3|H\rangle_4 \\
& = |\Phi^+\rangle_{13}|\Phi^+\rangle_{24} - |\Phi^-\rangle_{13}|\Phi^-\rangle_{24}
\end{aligned}$$

In the BSM, we only register those cases where both D2 and D4 detect a  $|+\rangle$  polarized photon for experimental simplicity, which consequently projects the photons 1 and 3 into the state  $|\Phi^+\rangle_{13}$ . Similarly we measure the QBER under a set of attenuation condition, which is also shown in Fig. 3 (square dots). The total equivalent attenuation rate this time is  $t = t_s^2 \times t_{a'}^4$ , which consists of two probabilistic PDC source and four attenuation plates (with a transmission rate  $t_{a'}$  ranging from 1 to 0.22) inserted in the light path as shown

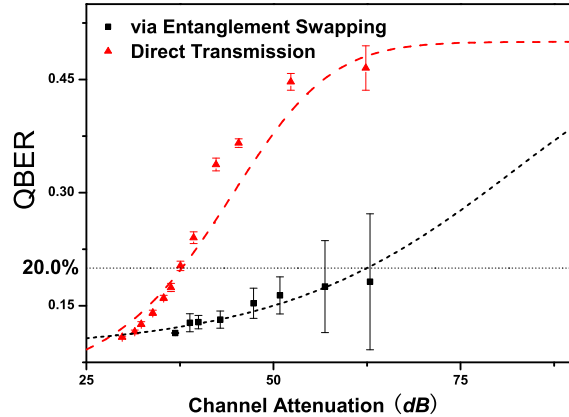


FIG. 3: Experimental results showing the advantage of quantum relay. The plots are measured QBERs at a certain equivalent distance. The shot dash line and dash line are numerical simulations according to the real parameters in our experiment with and without quantum delay, respectively.

in Fig. 2(b). We can see from the Fig. 3 that assisted with quantum relay the QBER arises significantly slower, demonstrating its superiority. The attenuation where the QBER reaches 20.0% this time is about 65dB.

Taking into account of the realistic experimental parameters: a success probability of 1/8 implementing BSM, an average probability of having a dark count in any single-photon detector  $D = 1.1 \times 10^{-4}$  (see [24] for definition), the average overall collection and detection efficiency  $\eta = 0.15$ , we follow the method developed in ref. [12] to carry out a numerical simulation [See Appendix] of the performance of our relay system, which is shown in Fig. 3 (dash lines). It can be seen from Fig. 3 that the experimental data shows a good agreement with the numerical simulation. Moreover, for the sake of clarity, we can transform the total attenuation into the equivalent distance of the communication channel with a loss of  $\alpha = 0.25\text{dB/km}$ , the typical photon loss in the telecom fiber channel. Thus, it is clearly shown that quantum relay extend the secure distance of quantum key distribution from 150km up to 260km.

In conclusion, by reasonable equivalences, we simulate an experiment of long distance quantum key distribution. We demonstrate the advantage of long-distance distribution of entangled photons assisted with quantum relay over the method of direct transmission. For

the first time we experimentally show that the distance of QC could be extended by making use of entanglement swapping, which is quite simple but powerful. Since entanglement swapping also serves as a key element in the scheme of quantum repeater, the experimental methods develop here may also be helpful for the future realization of quantum repeaters. Furthermore, by proper modifications, this experimental scheme allows an immediate experimental demonstration of third-man quantum cryptography and quantum secret sharing [26, 27]. While further practical investigations on real world are still necessary, we believe that this work provides a useful toolbox for tomorrow's long distance realization of QC.

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## Appendix: Analysis of QBER in Quantum-Relay-Assisted Key Distribution

As has been mentioned in the paper, in our experiment we aim to test the performance of quantum key distribution (QKD) over high photon loss channel with and without quantum relay. We examine under the scenario that the photons are emitted from a perfect entangled photon source (PEPS), that is, exactly one photon pair per pulse. Then we equivalently consider the actual probabilistic photon source produced by parametric down-conversion (PDC) as like the photons from the PEPS have passed a certain distance in a lossy channel, during which the photons are probabilistically lost. Similarly, additional attenuation plates placed into the light path are also used to simulate the distances. Sum up the above two effects and we get the total attenuation ( $t$ ).

In the experiment we use an ultraviolet pulsed laser from a mode-locked Ti:sapphire laser (pump power 470mW, center wavelength 394nm, pulse duration 200fs, repetition rate 76MHz). And we equivalently consider the entangled photon pair produced by PDC as the PEPS emitted with a frequency of  $76M/s$  passing through a quantum channel whose transmission probability is  $t_s$ . We observed in the experiment a two fold coincidence:  $c = 2.2 \times 10^4$  and an average detector efficiency  $\eta = 0.15$ . And thus  $t_s$  is

$$t_s = \frac{c/\eta^2}{76M} \quad (1)$$

Additional attenuation plates placed into the light path are also used to simulate the distances. The transmission probability of the channel with attenuation plates is  $t_c$ . The total transmission probability  $t$  is  $t = t_c \times t_s$ .

In our experiment, we measured an average count of  $20000s^{-1}$  under a moderate ambient lighting. The measurement time window is set to be  $5.5ns$  so the probability of having a dark count probability in one detector is:  $D = 1.1 \times 10^{-4}$ .

Following the method presented in Ref. [12], we perform a numerical simulation as follows:

First let us consider the case of direct QKD. For Alice and Bob to correctly receive and accept the signal, the photons must pass the channel unabsorbed ( $t$ ), and be detected by the detector ( $\eta$ ). So, the probability of existing an output at Alice's (Bob's) detectors in the time window is

$$P(Alice) = P(Bob) = [t^{\frac{1}{2}}\eta + (1 - t^{\frac{1}{2}}\eta)4D](1 - D)^3. \quad (2)$$

Here, in our experiment, we perform the symmetrical attenuation in the two channel of Alice and Bob, therefore, the attenuation of the each channel is  $t^{\frac{1}{2}}$ .

Thus the probability of sifted key bit is

$$\begin{aligned} P(total) &= \frac{1}{2} \cdot P(Alice) \cdot P(Bob) \\ &= \frac{1}{2} \{ [t^{\frac{1}{2}}\eta + (1 - t^{\frac{1}{2}}\eta)4D](1 - D)^3 \}^2. \end{aligned} \quad (3)$$

Because the unperfect futures of the source and the channel, in our experiment, we observe the optical visibility ( $V_{opt}$ ) of single photon is 0.95. Therefore, the probability of creating a correct key (signal) is,

$$P(signal) = \frac{1}{2} V_{2opt} [t^{\frac{1}{2}}\eta(1 - D)^3]^2, \quad (4)$$

$V_{2opt} = 0.95^2$  is the optical visibility of the photon pair.

When we perform the QKD via quantum relay, we perform the same attenuation in the four photon channel, therefore, the attenuation of the each channel is  $t^{\frac{1}{4}}$ . Similarly, in the time window the probability of Alice and Bob is

$$P_{QR}(Alice) = P_{QR}(Bob) = [t^{\frac{1}{4}}\eta + (1 - t^{\frac{1}{4}}\eta)4D](1 - D)^3. \quad (5)$$

In our experiment, the polarized beam-splitter and two single-photon detectors behind two polarizers setting at  $45^0$  constitute the Bell-state measurement (BSM) analyzer. The probability of that there is a BSM output  $P(Bell)$  is

$$P(Bell) = \frac{1}{2} (t^{\frac{1}{4}}\eta\eta_p)^2 + [(1 - t^{\frac{1}{4}}\eta\eta_p)D]^2 + 2t^{\frac{1}{4}}\eta\eta_p(1 - t^{\frac{1}{4}}\eta\eta_p)D, \quad (6)$$

here  $\eta_p = 0.5$  denotes the transmission probability of the polarizer in the BSM.  $\frac{1}{2}(t^{\frac{1}{4}}\eta\eta_p)^2$  is the probability of detecting one photon in each detector;  $[(1 - t^{\frac{1}{4}}\eta\eta_p)D]^2$  is the contribution from the dark count in both detectors; the probability of registering one photon in a detector while dark count in another is  $2t^{\frac{1}{4}}\eta\eta_p(1 - t^{\frac{1}{4}}\eta\eta_p)D$ .

Thus the probability of sifted key bit is

$$\begin{aligned} P_{QR}(total) &= P(Bell) \cdot \frac{1}{2} \cdot P_{QR}(Alice) \cdot P_{QR}(Bob) \\ &= \left\{ \frac{1}{2} (t^{\frac{1}{4}}\eta\eta_p)^2 + [(1 - t^{\frac{1}{4}}\eta\eta_p)D]^2 + 2t^{\frac{1}{4}}\eta\eta_p(1 - t^{\frac{1}{4}}\eta\eta_p)D \right\} \cdot \\ &\quad \frac{1}{2} [t^{\frac{1}{4}}\eta + (1 - t^{\frac{1}{4}}\eta)4D]^2 (1 - D)^6. \end{aligned} \quad (7)$$

The signal is

$$P_{QR}(signal) = V_{4opt} \left\{ \frac{1}{2} [t^{\frac{1}{4}} \eta (1 - D)^3]^2 \right\} \cdot \left[ \frac{1}{2} (t^{\frac{1}{4}} \eta \eta_p)^2 \right], \quad (8)$$

where  $V_{4opt} = 0.95^4$ .

$QBER$  of QKD is

$$QBER = \frac{1}{2} \left[ 1 - \frac{P(signal)}{P(total)} \right]. \quad (9)$$

When we perform the QKD via transmitting the entangled photon pairs directly,

$$QBER = \frac{1}{2} \left[ 1 - \frac{V_{2opt} [t^{\frac{1}{2}} \eta]^2}{[t^{\frac{1}{2}} \eta + (1 - t^{\frac{1}{2}} \eta) 4D]^2} \right]. \quad (10)$$

When we perform the QKD via quantum relay,

$$\begin{aligned} & QBER_{QR} \\ &= \frac{1}{2} \left[ 1 - \frac{V_{4opt} (t^{\frac{1}{4}} \eta)^4 \eta_p^2}{2 \left\{ \frac{1}{2} (t^{\frac{1}{4}} \eta \eta_p)^2 + [(1 - t^{\frac{1}{4}} \eta \eta_p) D]^2 + 2 t^{\frac{1}{4}} \eta \eta_p (1 - t^{\frac{1}{4}} \eta \eta_p) D \right\} \cdot [t^{\frac{1}{4}} \eta \eta_p + (1 - t^{\frac{1}{4}} \eta) 4D]^2} \right]. \end{aligned} \quad (11)$$